On Superposed Couple-stress Fluids in Porous Medium in Hydromagnetics

Sunil[a], R. C. Sharma[b], and R. S. Chandel[c]

[a] Department of Applied Sciences, National Institute of Technology, Hamirpur (H.P.) – 177005, India
[b] Department of Mathematics, Himachal Pradesh University, Summer Hill, Shimla – 171005, India
[c] Department of Mathematics, Government Degree College, Palampur (H.P.) – 176061, India

Reprint requests to Dr. Sunil

Z. Naturforsch. **57a,** 955 – 960 (2002); received August 22, 2001

The Rayleigh-Taylor instability of two superposed couple-stress fluids of uniform densities in a porous medium in the presence of a uniform horizontal magnetic field is studied. For mathematical simplicity, the stability analysis is carried out for two highly viscous fluids of equal kinematic viscosity and equal couple-stress kinematic viscosity. A potentially stable configuration remains stable under certain conditions, and a potentially unstable configuration is stable under certain conditions. The magnetic field stabilizes a certain wave-number range $k > k^*$, which is unstable in the absence of the magnetic field.

Key words: Couple-stress Fluids; Borous Medium; Magnetic Field.

1. Introduction

The instability of a plane interface between two incompressible viscous fluids of different densities, when the lighter one is accelerated into the heavier one, has been discussed by Chandrasekhar [1]. Bhatia [2] has studied the influence of the viscosity on the stability of a plane interface separating two incompressible superposed conducting fluids of uniform density, when the whole system is in a uniform magnetic field. He has carried out the stability analysis for two highly viscous fluids of equal kinematic viscosity and different uniform densities. The Rayleigh-Taylor instability of two viscoelastic (Oldroyd) superposed fluids has been studied by Sharma and Sharma [3].

With the growing importance of non-Newtonian fluids in modern technology, the investigation of such fluids is desirable. Stokes [4] has formulated the theory of a couple-stress fluid. The presence of small amounts of additives in a lubricant can improve the bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. These additives in a lubricant also reduce the coefficient of friction and increase the temperature range in which the bearing can operate.

A number of theories of the microcontinuum has been proposed and applied (Stokes [4]; Lai et al. [5]; Walicka [6]). The theory of Stokes [4] allows for polar effects such as the presence of couple-stresses and body couples and has been applied to the study of some simple lubrication problems (see e.g. Sinha et al. [7]; Bujurke and Jayaraman [8], Lin [9]). According to [4], couple-stresses appear in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [10] modeled synovial fluids as couple-stress fluids in human joints. Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [11] has studied the effect of a horizontal magnetic field, which varies in the vertical direction, on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of a magnetic field the system is known to be stable. In all the above studies, the medium has been considered to be non-porous.

In recent years, the investigation of the flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the books by Phillips [12], Ingham and Pop [13], and Nield and Bejan [14]. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous and couple-stress viscous terms in the

0932-0784 / 02 / 1200–0955 $ 6.00 © Verlag der Zeitschrift für Naturforschung, Tübingen · www.znaturforsch.com

equation of couple-stress fluid motion are replaced by the resistance term $\left[-\frac{1}{k_1}(\mu-\mu'\nabla^2)\boldsymbol{q}\right]$, where μ and μ' are the viscosity and couple-stress viscosity, k_1 is the medium permeability and $\boldsymbol{q}$ the Darcian (filter) velocity of the couple-stress fluid. Recently, the thermal instability of a couple-stress fluid in a porous medium in the presence of rotation and magnetic field, separately, has been studied by Sharma et al. [15] and Sharma and Thakur [16].

Keeping in mind the importance and applications of non-Newtonian fluids, porous medium and magnetic field; the present paper attempts to study the hydromagnetics of superposed couple-stress fluids in a porous medium.

2. Formulation of the Problem and Perturbation Equations

We consider an incompressible couple-stress fluid layer, consisting of an infinitely conducting (hydromagnetic) fluid of density ρ, arranged in horizontal strata and acted on by a horizontal magnetic field $\boldsymbol{H}(H, 0, 0)$ and a gravity field $\boldsymbol{g}(0, 0, -g)$. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

The hydromagnetic equations governing the motion of a couple-stress fluid through a porous medium in hydromagnetics are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right]=-\nabla p+\rho\boldsymbol{g}-\frac{\rho}{k_1}\times\left(\nu-\nu'\nabla^2\right)\boldsymbol{q}+\frac{\mu_e}{4\pi}(\nabla\times\boldsymbol{H})\times\boldsymbol{H}, \tag{1}$$

$$\nabla\cdot\boldsymbol{q}=0, \tag{2}$$

$$\left(\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}\cdot\nabla\right)\rho=0, \tag{3}$$

$$\nabla\cdot\boldsymbol{H}=0, \tag{4}$$

$$\varepsilon\frac{\partial \boldsymbol{H}}{\partial t}=(\boldsymbol{H}\cdot\nabla)\boldsymbol{q}-(\boldsymbol{q}\cdot\nabla)\boldsymbol{H}, \tag{5}$$

where ρ, p and $\boldsymbol{q}(u, v, w)$ are the density, pressure, and filter velocity of a hydromagnetic fluid, respectively. ν, ν' and μ_e stand for the kinematic viscosity, couple-stress kinematic viscosity and the magnetic permeability, respectively. Eqs. (1) and (2) represent the equations of motion and continuity for the couple-stress fluid, whereas (3) represents the fact that the density of a fluid particle remains unchanged as we follow it with its motion, whereas (4) and (5) are Maxwell's equations.

The initial stationary state, whose stability we wish to examine, is that of an incompressible couple-stress fluid of variable densitiy, viscosity and couple-stress viscosity, arranged in horizontal strata in a porous medium. The system is acted on by a horizontal magnetic field $\boldsymbol{H}(H, 0, 0)$. Consider an infinite horizontal layer of thickness d bounded by the planes at $z=0$ and $z=d$. The character of the equilibrium of this initial static state is determined, as usual, by supposing that the system is slightly disturbed and by following its further evolution.

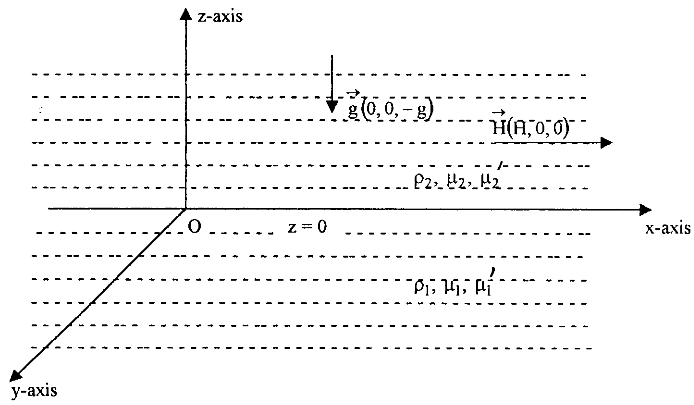

Fig. 1. Two superposed couple-stress fluids in a porous medium in hydromagnetics.

Let $\delta\rho$, δp, $\boldsymbol{q}$ (u, v, w) and $\boldsymbol{h}(h_x, h_y, h_z)$ denote, respectively, the perturbations in hydromagnetic fluid density $\rho(z)$, pressure $p(z)$, velocity (0, 0, 0) and magnetic field $(H, 0, 0)$. Then the linearized perturbation equations governing the motion of the couple-stress fluid through the porous medium are

$$\frac{\rho}{\varepsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla\delta p + \boldsymbol{g}\delta\rho - \frac{\rho}{k_1}\left(\nu - \nu'\nabla^2\right)\boldsymbol{q} + \frac{\mu_e}{4\pi}(\nabla\times\boldsymbol{h})\times\boldsymbol{H}, \tag{6}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{7}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho = -wD\rho, \tag{8}$$

$$\nabla\cdot\boldsymbol{h} = 0, \tag{9}$$

$$\varepsilon\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H}\cdot\nabla)\boldsymbol{q}. \tag{10}$$

Analyzing the perturbations into normal modes, we assume that the perturbed quantities have a space and time dependence of the form

$$f(z)\exp(ik_x x + ik_y y + nt), \tag{11}$$

where k_x, k_y are the wave numbers along the x- and y-direction, respectively, $k = \sqrt{\left(k_x^2 + k_y^2\right)}$ is the resultant wave number of disurbance, n the growth rate of the harmonic disturbance which is, in general, a complex constant, and $f(z)$ is some function of z.

For perturbations of the form (11), (6) – (10) become

$$\left[\frac{n}{\varepsilon} + \frac{1}{k_1}\left\{\nu - \nu'\left(D^2 - k^2\right)\right\}\right]\rho u = -ik_x\delta p, \tag{12}$$

$$\left[\frac{n}{\varepsilon} + \frac{1}{k_1}\left\{\nu - \nu'\left(D^2 - k^2\right)\right\}\right]\rho v = -ik_y\delta p + \frac{\mu_e H}{4\pi}\left(ik_x h_y - ik_y h_x\right), \tag{13}$$

$$\left[\frac{n}{\varepsilon} + \frac{1}{k_1}\left\{\nu - \nu'\left(D^2 - k^2\right)\right\}\right]\rho w = -D\delta\rho - g\delta\rho + \frac{\mu_e H}{4\pi}(ik_x h_z - Dh_x), \tag{14}$$

$$ik_x u + ik_y + Dw = 0, \tag{15}$$

$$\varepsilon n\delta\rho = -wD\rho, \tag{16}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \tag{17}$$

$$\varepsilon nh_x = ik_x Hu,\ \varepsilon nh_y = ik_x Hv,\ \varepsilon nh_z = ik_x Hw, \tag{18}$$

where $D = \dfrac{\mathrm{d}z}{\mathrm{d}}$.

Eliminating δp between Eqs. (12) – (14) and using Eqs. (15) – (18), we obtain

$$\frac{n}{\varepsilon}\left[D(\rho Dw) - k^2\rho w\right] + \frac{gk^2}{\varepsilon n}(D\rho)w + \frac{1}{k_1} \times\left[D\left\{\mu - \mu'\left(D^2 - k^2\right)\right\}Dw - k^2\left\{\mu - \mu'\left(D^2 - k^2\right)\right\}w\right] + \frac{\mu_e H^2 k_x^2}{4\pi\varepsilon n}\left(D^2 - k^2\right)w = 0. \tag{19}$$

3. Two Uniform Couple-stress Fluids Separated by a Horizontal Boundary

We consider the case when two superposed couple-stress fluids of uniform densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 and uniform couple-stress viscosities $\mu_1{}'$ and $\mu_2{}'$ are separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and the upper couple-stress fluids, respectively. Then, in each region of constant ρ, constant μ and constant μ', (19) becomes

$$\left(D^2 - k^2\right)\left(D^2 - q^2\right)w = 0, \tag{20}$$

where

$$q^2 = k^2 + \frac{\nu}{\nu'} + \frac{nk_1}{\varepsilon\nu'} + \frac{\mu_e H^2}{4\pi}\frac{k_1 k_x^2}{\mu'\varepsilon n}.$$

The general solution of (20) is a linear combination of the solutions

$$e^{\pm kz} \text{ and } e^{\pm qz}. \tag{21}$$

Since w must vanish both when $z \to -\infty$ (in the lower fluid) and $z \to +\infty$ (in the upper fluid), the general solution of (20) can be written as

$$w_1 = A_1 e^{+kz} + B_1 e^{+q_1 z}, (z < 0), \tag{22}$$

$$w_2 = A_2 e^{-kz} + B_2 e^{-q_2 z}, (z > 0), \tag{23}$$

where A_1, B_1, A_2, B_2 are constants of integration,

$$q_1 = \sqrt{\left(k^2 + \frac{\nu_1}{\nu_1'} + \frac{nk_1}{\varepsilon\nu_1'} + \frac{\mu_e H^2}{4\pi}\frac{k_1 k_x^2}{\rho_1 \nu_1' \varepsilon n}\right)}$$

and

$$q_2 = \sqrt{\left(k^2 + \frac{\nu_2}{\nu_2'} + \frac{nk_1}{\varepsilon\nu_2'} + \frac{\mu_e H_2}{4\pi}\frac{k_1 k_x^2}{\rho_2 \nu_2' \varepsilon n}\right)}. \tag{24}$$

In writing the solutions (22) and (23), it is assumed that q_1 and q_2 are so defined that their real parts are positive. The solutions (22) and (23) must satisfy certain boundary conditions. The boundary conditions to be satisfied at the inface $z = 0$ are that

$$w \tag{25}$$

and

$$Dw \tag{26}$$

must be continuous.

The constitutive equations for the couple-stress fluid are

$$\tau_{ij} = \left(2\mu - 2\mu'\nabla^2\right)e_{ij}, \quad e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{27}$$

The conditions on a free surface are that

$$\begin{aligned}\tau_{xz} &= \left(\mu - \mu'\nabla^2\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\\ &= \left[\mu - \mu'\left(D^2 - k^2\right)\right](Du + ik_x w)\end{aligned} \tag{28}$$

and

$$\begin{aligned}\tau_{yz} &= \left(\mu - \mu'\nabla^2\right)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\\ &= \left[\mu - \mu'\left(D^2 - k^2\right)\right]\left(Dv + ik_y w\right)\end{aligned} \tag{29}$$

must be continuous. Therefore $ik_x\tau_{xz} + ik_y\tau_{yz}$, with the help of (15), yields that

$$\left[\mu - \mu'\left(D^2 - k^2\right)\right]\left(D^2 + k^2\right)w \tag{30}$$

must be continuous at the interface $z = 0$.

Integrating (19) across the interface $z = 0$, we obtain another condition:

$$\begin{aligned}&[\rho_2 Dw_2 - \rho_1 Dw_1]_{z=0} + \frac{\varepsilon}{nk_1}[\{\mu_2 - \mu_2'(D^2 - k^2)\}Dw_2\\ &- \{\mu_1 - \mu_1'(D^2 - k^2)\}Dw_1]_{z=0}\\ &= -\frac{\mu_e H^2}{4\pi}\frac{k_x^2}{n^2}(Dw_2 - Dw_1)_{z=0} - \frac{gk^2}{n^2}(\rho_2 - \rho_1)w_0,\end{aligned} \tag{31}$$

where w_0 is the common value of w_1 and w_2 at $z = 0$.

Applying the boundary conditions (25), (26), (30) and (31) to the solutions (22) and (23), we abtain

$$A_1 + B_1 = A_2 + B_2 (= w_0), \tag{32}$$

$$kA_1 + q_1 B_1 = -kA_2 - q_2 B_2 (= Dw_0), \tag{33}$$

$$\begin{aligned}&[\mu_1\{2k^2 A_1 + (q_1^2 + k^2)B_1\} - \mu_1'(q_1^2 - k^2)(q_1^2 + k^2)B_1]\\ &= [\mu_2\{2k^2 A_2 + (q_2^2 + k^2)B_2\} - \mu_2'(q_2^2 - k^2)\\ &\quad (q_2^2 + k^2)B_2]\\ &(= \{\mu - \mu'(D^2 - k^2)\}(D^2 + k^2)w_0),\end{aligned} \tag{34}$$

$$\begin{aligned}&[-k\rho_2 A_2 - q_2\rho_2 B_2 - k\rho_1 A_1 - q_1\rho_1 B_1]\\ &+ \frac{gk^2}{2n^2}(\rho_2 - \rho_1)[A_1 + B_1 + A_2 + B_2]\\ &+ \frac{\varepsilon}{nk_1}[\mu_2(-kA_2 - q_2 B_2) - \mu_1(kA_1 + q_1 B_1)\\ &+ \mu_2'(q_2^2 - k^2)(q_2)B_2 + \mu_1'(q_1^2 - k^2)q_1 B_1]\\ &+ \frac{\mu_e H^2}{4\pi}\frac{k_x^2}{n^2}(-kA_2 - q_2 B_2 - kA_1 - q_1 B_1) = 0.\end{aligned} \tag{35}$$

Eliminating the constants A_1, B_1, A_2, B_2 from (32)–(35), we obtain a fourth order determinant. This determinant can be reduced to a third order determinant by subtracting the first column from the second, the third column from the fourth and adding the first column to the third. Removing the factor $q_1 - k$ and $q_2 - k$ (as $q_1 = k$ and $q_2 = k$ are two characteristic roots which lead to trivial solutions) and expanding the remaining determinant, we obtain the following characteristic equation:

$$\begin{aligned}
&\{2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\} \\
&\quad\times\left\{-\frac{\alpha_2}{k}\left(1+\frac{\varepsilon\nu_2}{nk_1}\right)+\frac{\varepsilon}{nk_1}\frac{\alpha_2\nu_2'q_2}{k}(q_2+k)-\frac{V_A^2k_x^2}{n^2k}\right\} \\
&\quad-\left\{R-1-\frac{\varepsilon}{nk_1}(\alpha_1\nu_1+\alpha_2\nu_2)-\frac{2V_A^2k_x^2}{n^2}\right\} \\
&\quad\times\left\{-\alpha_2\nu_2(q_2+k)+\alpha_2\nu_2'(q_2+k)\left(q_2^2+k^2\right)\right\} \\
&\quad+2k\Bigg[\left\{-\alpha_2\nu_2(q_2+k)+\alpha_2\nu_2'(q_2+k)\left(q_2^2+k^2\right)\right\} \\
&\quad\times\left\{-\frac{\alpha_1}{k}\left(1+\frac{\varepsilon\nu_1}{nk_1}\right)+\frac{\varepsilon}{nk_1}\frac{\alpha_1\nu_1'q_1}{k}(q_1+k)-\frac{V_A^2k_x^2}{n^2k}\right\} \\
&\quad-\left\{\alpha_1\nu_1(q_1+k)-\alpha_1\nu_1'(q_1+k)\left(q_1^2+k^2\right)\right\} \\
&\quad\times\left\{-\frac{\alpha_2}{k}\left(1+\frac{\varepsilon\nu_2}{nk_1}\right)+\frac{\varepsilon}{nk_1}\frac{\alpha_2\nu_2'q_2}{k}(q_2+k)-\frac{V_A^2k_x^2}{n^2k}\right\}\Bigg] \\
&\quad+\left\{\alpha_1\nu_1(q_1+k)-\alpha_1\nu_1'(q_1+k)\left(q_1^2+k^2\right)\right\} \\
&\quad\times\left\{R-1-\frac{\varepsilon}{nk_1}(\alpha_1\nu_1+\alpha_2\nu_2)-\frac{2V_A^2k_x^2}{n^2}\right\} \\
&\quad-\left\{2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\right\} \\
&\quad\times\left\{-\frac{\alpha_1}{k}\left(1+\frac{\varepsilon\nu_1}{nk_1}\right)+\frac{\varepsilon}{nk_1}\frac{\alpha_1\nu_1'q_1}{k}(q_1+k)-\frac{V_A^2k_x^2}{n^2k}\right\} \\
&=0, \qquad (36)
\end{aligned}$$

where

$$\alpha_{1,2}=\frac{\rho_{1,2}}{\rho_1+\rho_2},\ \alpha_2=\frac{\rho_2}{\rho_1+\rho_2},$$
$$R=\frac{gk}{n^2}(\alpha_2-\alpha_1),\quad V_A^2=\frac{\mu_e H^2}{4\pi(\rho_1+\rho_1)}.$$

4. Discussion

The dispersion relation (36) is quite complicated, as the values of q_1 and q_2 involve square roots. We therefore make the assumption that the couple-stress kinematic viscosity ν' is very high.

Under this assumption we have

$$\begin{aligned}
q&=k\left[1+\frac{\nu}{\nu'k^2}+\frac{nk_1}{\varepsilon\nu'k^2}+\frac{\mu_eH^2}{4\pi}\frac{k_1k_x^2}{\mu'k^2\varepsilon n}\right]^{1/2} \\
&=k\left[1+\frac{\nu}{2\nu'k^2}+\frac{nk_1}{2\varepsilon\nu'k^2}+\frac{\mu_eH^2}{8\pi}\frac{k_1k_x^2}{\mu'k^2\varepsilon n}\right]. \qquad (37)
\end{aligned}$$

Putting $\nu_1=\nu_2=\nu$, $\nu_1{}'=\nu_2{}'=\nu'$ (the case of equal kinematic viscosities and equal couple-stress kinematic viscosities), for mathematical simplicity, but any of the essential features of the problem would not be obscured by this simplifying assumption, we have

$$q_1+k=q_2+k=\left[2k+\frac{\nu}{2\nu'k}+\frac{nk_1}{2\varepsilon\nu'k}+\frac{\mu_eH^2}{8\pi}\frac{k_1k_x^2}{\mu'k\varepsilon n}\right] \qquad (38)$$

and

$$\begin{aligned}
q_1^2+k^2&=q_2^2+k^2 \\
&=\left[2k^2+\frac{\nu}{\nu'}+\frac{nk_1}{\varepsilon\nu'}+\frac{\mu_eH^2}{4\pi}\frac{k_1k_x^2}{\mu'\varepsilon n}\right]. \qquad (39)
\end{aligned}$$

Substituting the values of $q_1+k, q_2+k, q_1^2+k^2$ and $q_2^2+k^2$ from (38) and (39) in (36), we obtain the dispersion relation

$$\begin{aligned}
&A_8n^8+A_7n^7+A_6n^4+A_5n^5+A_4n^4 \\
&\quad+A_3n^3+A_2n^2+A_1n+A_0 \\
&\quad=0, \qquad (40)
\end{aligned}$$

where

$$A_8=[\varepsilon\nu'(P_1P_6)],\ \ A_7=[\varepsilon\nu'(P_1P_7+P_2P_6)+k_1(S_1X_8)],$$

$$A_6=[\varepsilon\nu'(P_1P_8+P_2P_7+P_3P_6)+k_1(S_1X_9+S_2X_8)],$$

$$\begin{aligned}A_5=[&\varepsilon\nu'(P_1P_9+P_2P_8+P_3P_7+P_4P_6)\\&+k_1(S_1X_{10}+S_2X_9+S_3X_8)+S_6],\end{aligned}$$

$$\begin{aligned}A_4=[&\varepsilon\nu'(P_1P_{10}+P_2P_9+P_3P_8+P_4P_7+P_5P_6)\\&+k_1(S_2X_{10}+S_3X_9+S_4X_8)+S_7],\end{aligned}$$

$$\begin{aligned}A_3=[&\varepsilon\nu'(P_2P_{10}+P_3P_9+P_4P_8+P_5P_7)\\&+k_1(S_3X_{10}+S_4X_9+S_5X_8)],\end{aligned}$$

$$A_2=[\varepsilon\nu'(P_3P_{10}+P_4P_9+P_5P_8)+k_1(S_4X_{10}+S_5X_9)],$$

$$A_1=[\varepsilon\nu'(P_4P_{10}+P_5P_9)+k_1(S_5X_{10})],\ A_0=[\varepsilon\nu'(P_5P_{10})],$$

and

$$\begin{aligned}
&P_1=X_1^2,\ P_2=X_1(X_4+X_2),\ P_3=(2X_1X_3+X_2X_4),\\
&P_4=X_3(X_2+X_4),\ P_5=X_2^3,\ P_6=4\alpha_1\alpha_2(X_1X_5),\\
&P_7=4\alpha_1\alpha_2(X_1X_6+X_4X_5),\\
&P_8=4\alpha_1\alpha_2\left(X_1X_7+X_3X_5+X_4X_6-\frac{2k\nu(\alpha_1-\alpha_2)^2}{4\alpha_1\alpha_2}\right),
\end{aligned}$$

$$P_9 = X_3X_6 + X_4X_7, P_{10} = X_3X_7,$$
$$S_1 = X_1X_5,$$
$$S_2 = (X_1X_6 + X_4X_5),$$
$$S_3 = (X_1X_7 + X_4X_6 + X_3X_5), S_4 = (X_4X_7 + X_3X_6),$$
$$S_5 = X_3X_7, S_6 = 2kk_1\nu(\alpha_1 - \alpha_2)^2, S_7 = 2\varepsilon k\nu^2(\alpha_1 - \alpha_2)^2,$$
$$X_1 = \frac{k_1}{2\varepsilon\nu' k}, X_2 = \left(k + \frac{\nu}{2\nu' k}\right), X_3 = \left(\frac{\mu_e H^2}{8\pi}\frac{k_1 k_x^2}{\mu'\varepsilon k}\right),$$
$$X_4 = \left(2k + \frac{\nu}{2\nu' k}\right), \quad X_5 = \left(\frac{k_1}{\varepsilon}\right),$$
$$X_6 = (2\nu' k^2), X_7 = \left(\frac{\mu_e H^2}{4\pi}\frac{k_1 k_x^2 \nu'}{\mu'\varepsilon}\right),$$
$$X_8 = (\alpha_1 - \alpha_2)^2, X_9 = (\alpha_1 - \alpha_2)^2\frac{\varepsilon\nu}{k_1},$$
$$X_{10} = gk(\alpha_1 - \alpha_2). \tag{41}$$

(a) Stable case:

For the potentially stable arrangement $\alpha_1 > \alpha_2$ we find, by applying Hurwitz's criterion to (40), that all the coefficients in (40) are positive if

$$\alpha_1\alpha_2 > \frac{1}{6}, \tag{42}$$

and so all the roots 'n' are either real and negative; or there are complex roots (which occur in pairs) with negative real parts and rest negative real roots. The system is, therefore, stable in each case. Hence, the potentially stable configuration remains stable for the case of two superposed couple-stress fluids in a porous medium in hydromagnetics under the condition $\alpha_1\alpha_2 > \frac{1}{6}$.

(b) Unstable case:

For the potentially unstable arrangement $\alpha_2 > \alpha_1$, all the coefficients in (40) are positive if

$$\alpha_1\alpha_2 > \frac{1}{6} \text{ and } k_x^2 V_A^2 > 2gk(\alpha_1 - \alpha_2), \tag{43}$$

where $V_A^2 = \frac{\mu_e H^2}{4\pi\rho}$, and so all the roots '$n$' are either real and negative; or there are complex roots (which occur in pairs) with negative real parts and the rest negative real roots. The system is, therefore, stable in each case.

Hence for the stability of two superposed, couple-stress fluids in a porous medium in hydromagnetics, besides $\alpha_1\alpha_2 > \frac{1}{6}$, we have $k_x^2 V_A^2 > 2gk(\alpha_1 - \alpha_2)$

i.e. $k > \frac{2g(\alpha_1 - \alpha_2)}{V_A^2}\sec^2\theta (= k*)$, where $V_A^2 = \frac{\mu_e H^2}{4\pi\rho}$ and θ is the angle between the wave vector $\boldsymbol{k}$ and the magnetic field $\boldsymbol{H}$. The magnetic field therefore stabilizes a certain wave-number range $k > k^*$, which would be unstable in the absence of a magnetic field (Sharma et al. [17]).

Acknowledgement

The financial assistance to Dr. Sunil in the form of Research and Development Project [No. 25(0129)/02/EMR-II] of the Council of Scientific and Industrial Research (CSIR), New Delhi is gratefully acknowledged.

[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York 1981.
[2] P. K. Bhatia, Nuovo Cim. **19B**, 161 (1974).
[3] R. C. Sharma and K. C. Sharma, Acta Physica Hungarica **45**, 213 (1978).
[4] V. K. Stokes, Phys. Fluids **9**, 1709 (1966).
[5] W. M. Lai, S. C. Kuei, and V. C. Mow, J. Biomechanical Engin. **100**, 169 (1978).
[6] A. Walicka, Micropolar Flow in a Slot between Rotating Surfaces, Zielona Góra, TU Press, 1994.
[7] P. Sinha, C. Singh, and K. R. Prasad, Wear **67**, 15 (1981).
[8] N. M. Bujurke and G. Jayaraman, Int. J. Mech. Sci. **24**, 369 (1982).
[9] J. R. Lin, Appl. Mech. Engin. **1**, 317 (1996).
[10] E. Walicki and A. Walicka, Appl. Mech. Engin. **4**, 363 (1999).
[11] A. Kent, Phys. Fluids **9**, 1286 (1966).
[12] O. M. Phillips, Flow and Reaction in Permeable Rocks, Cambridge University Press, Cambridge 1991.
[13] D. B. Ingham and I. Pop, Transport Phenomena in Porous Medium, Pergamon, Oxford 1998.
[14] D. A. Nield and A. Bejan, Convection in Porous Medium, 2nd ed., Springer-Verlag, New York 1999.
[15] R. C. Sharma, Sunil, and M. Pal, Appl. Mech. Engin. **5(4)**, 883 (2000).
[16] R. C. Sharma and K. D. Thakur, Czech. J. Phys. **50**, 753 (2000).
[17] R. C. Sharma, Sunil, and M. Pal, Studia Geotechnica et Mechanica **3–4**, 55 (2001).